An Accurate Model for Pull-in Voltage of Circular Diaphragm Capacitive Micromachined Ultrasonic Transducers (CMUT)

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Research Objective

- Develop an accurate Analytical Model to calculate Pull-in Voltage of a CMUT device built with Circular diaphragm.

  CMUT devices are a type of MEMS capacitive ultrasonic sensors characterized by small device dimensions.

  The model takes into account

  - *Non-linear stretching of the diaphragm due to Large Deflection,*
  - *Built-in Residual Stress of the membrane,*
  - *Bending Stress, and*
  - *the Fringing Field effect*, and
  - *the non-linearity of the Electrostatic Field*

Presentation Outline

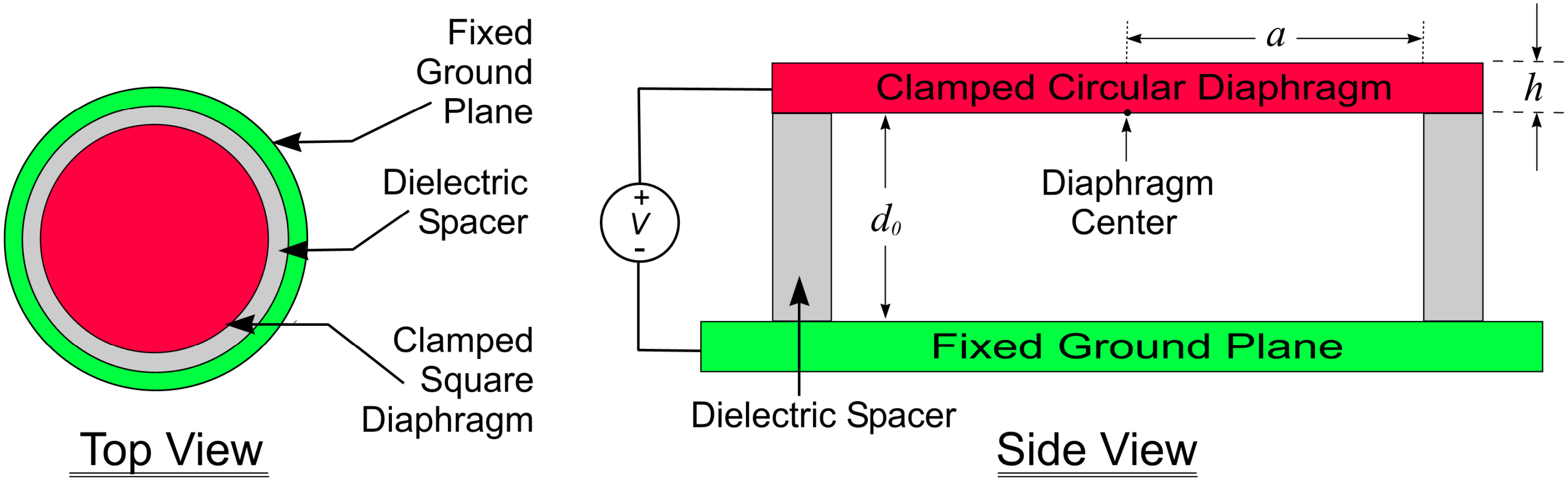
- Motivation
- Basic Device Structure and Operation
- Limitations in Existing Models
- Solution Approach
- Model Development
- Model Validation
- Conclusions

Motivation

- CMUT devices has
  - Wide Range of Applications
    - Biomedical Imaging
    - Automotive Collision Avoidance Radar System
    - Nondestructive Testing
    - Microphones
  - Superior performance over traditional piezoelectric transducers
    - low temperature sensitivity
    - high signal sensitivity
    - wide bandwidth
    - very high level of integration (low cost)

Estimating Pull-in Voltage $V_{\text{Pull-in}}$ and the Plate Travel Distance x_P before pull-in effect is required for the successful design of electrostatic actuators, switches, varactors, and sensors.

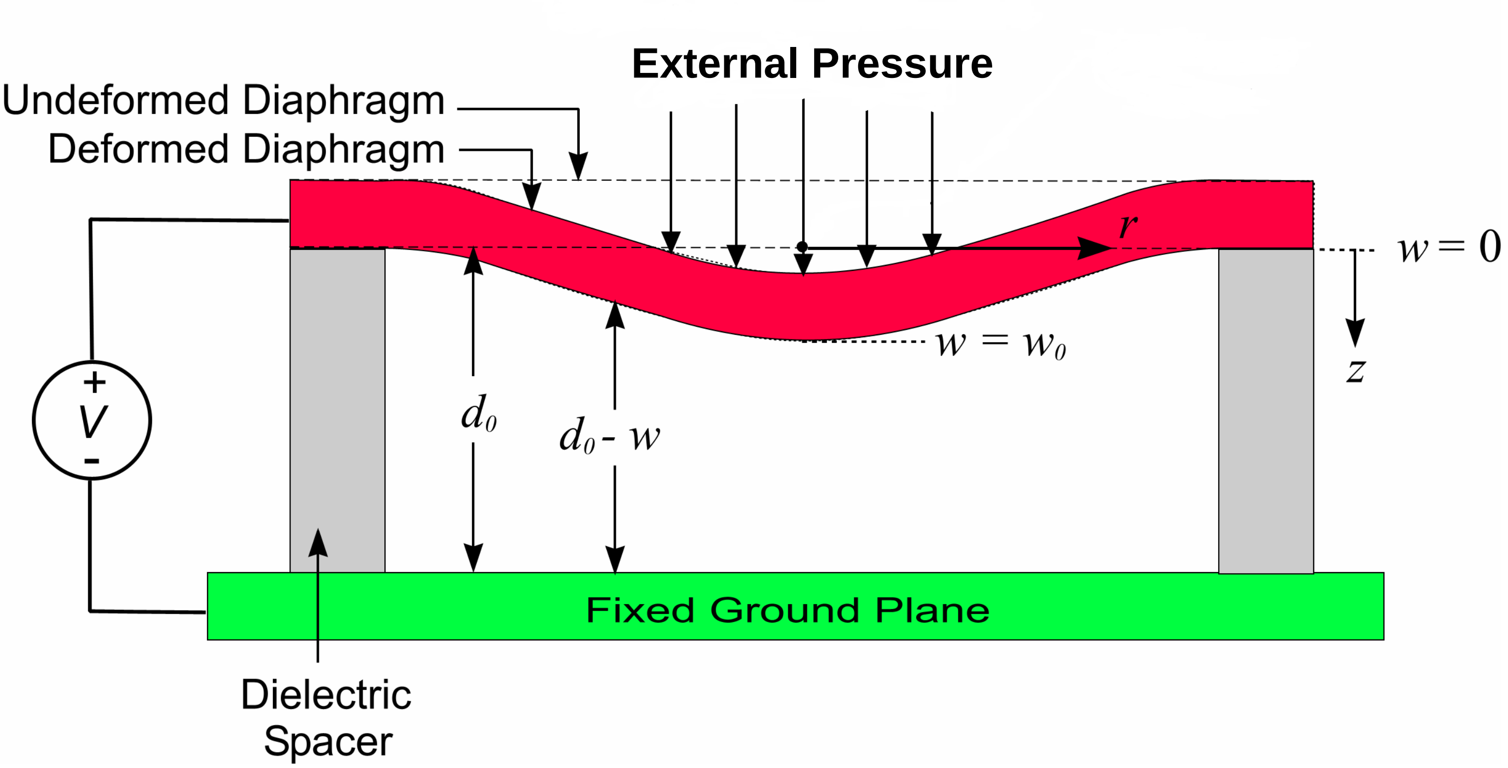
Basic Device Structure



Basic structure of a CMUT device with a circular diaphragm.

Operation Principle

Diaphragm deformation after subject to an external pressure.



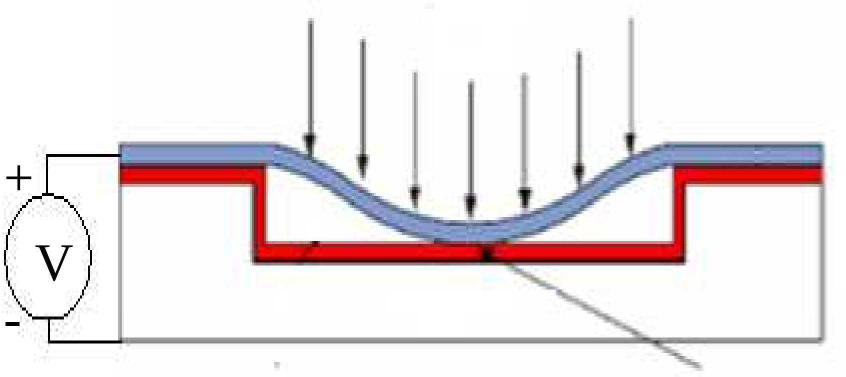
- *At equilibrium, Electrostatic Attraction Force is balanced by the Elastic Restoring Force of the Diaphragm*

Diaphragm Collapse and Pull-in Voltage ($V_{\text{Pull-in}}$)

For $V < V_{\text{Pull-in}}$, *Electrostatic Force = Elastic Restoring Force*

For $V > V_{\text{Pull-in}}$, *Electrostatic Force > Elastic Restoring Force*

Diaphragm collapses under excessive electrostatic pressure



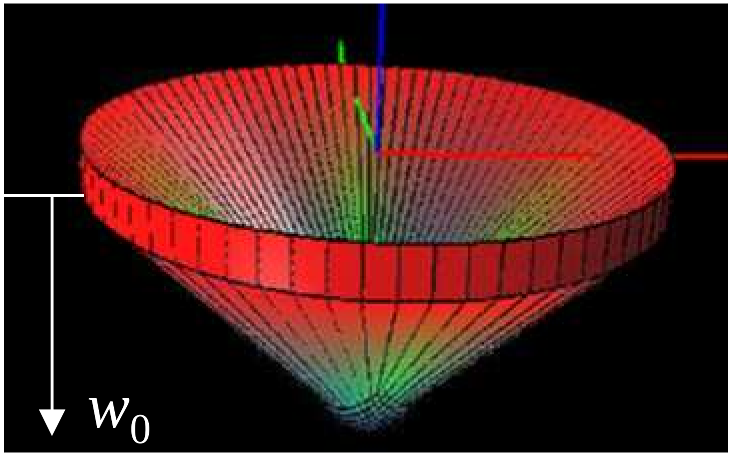
Collapsed Diaphragm

Limitations in Existing Models: Parallel Plate Approximation

- Existing models are based on **parallel plate approximation**
- Assumes piston-like motion of the diaphragm and predicts pull-in when center deflection reaches ⅓ of the airgap

$$w_{0-\mathrm{PI}} = 1/3d_0 \quad (1)$$

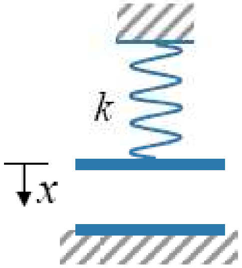
Pull-in travel of the diaphragm



Deflection of Circular Diaphragm in Intellisuite

Limitations in Existing Models: Spring Hardening Effect

- Does not account for **spring hardening effect** due to non-linear stretching of the diaphragm under **large deflection,** more pronounced in **thin diaphragms** as in CMUTs



Force $F = kx$, k is the spring constant

Not valid for large deflection

- Spring hardening causes the Pull-in Voltage to go up and center deflection can be as high as 50% of the airgap

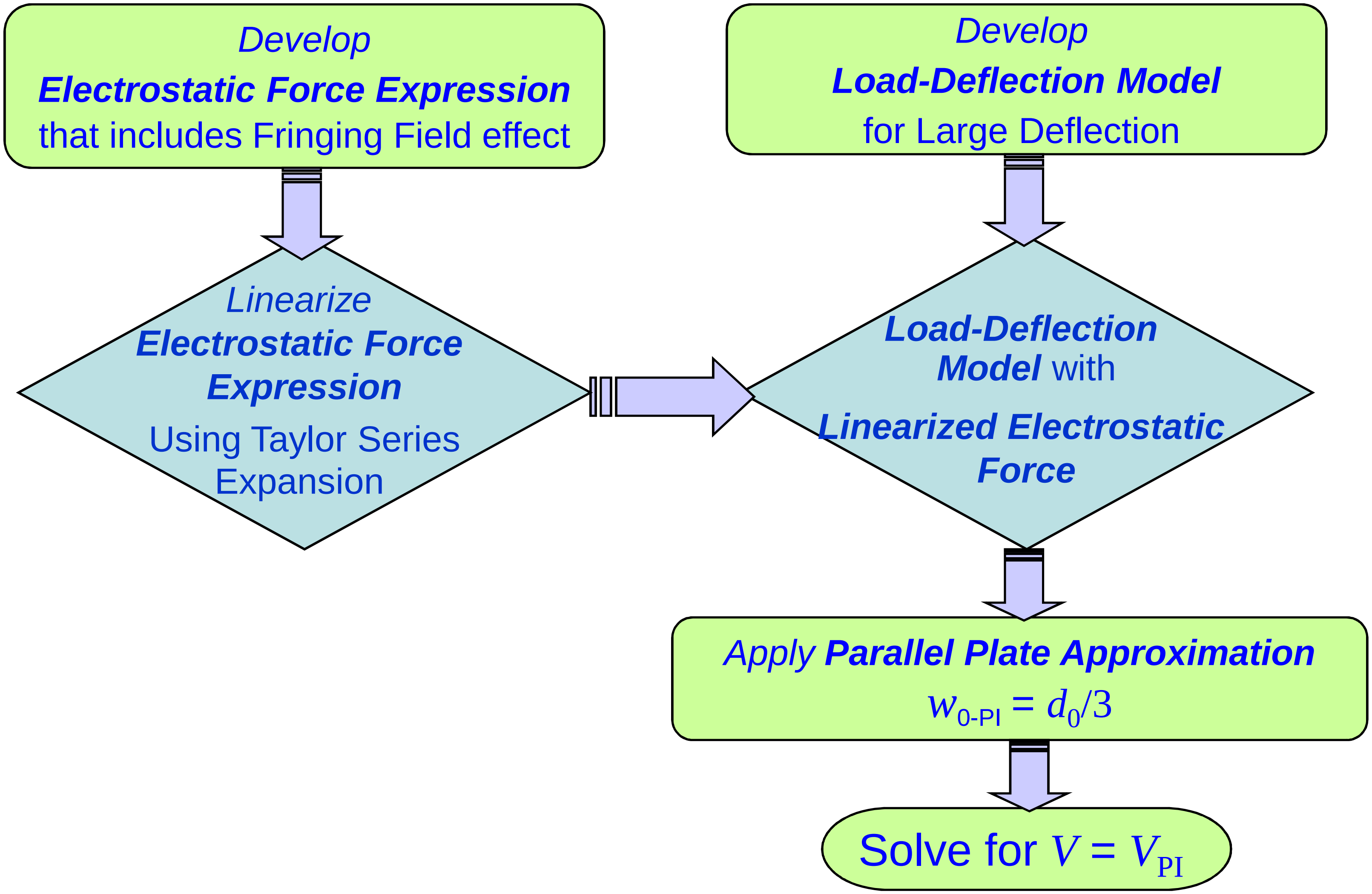
$$w_{0-\mathrm{PI}} \approx 0.5 d_0 \quad (2)$$

- Does not take into account the effect of **Poisson ratio**

Limitations in Existing Models: Fringing Field Effect

- Existing model does take into account the effect due to **Fringing Fields**
- Fringing Field effect causes **spring softening effect** of the diaphragm, lowering the pull-in voltage
- The effect is more pronounced in sensors with small diaphragm, such as in CMUT devices
- **Above simplifications results in an error as high as 20% when compared to experimental and FEA results**

Solution Approach



Develop
Electrostatic Force Expression
that includes Fringing Field effect

Linearize
Electrostatic Force Expression
Using Taylor Series Expansion

Develop
Load-Deflection Model
for Large Deflection

Load-Deflection Model with
Linearized Electrostatic Force

Apply ***Parallel Plate Approximation***
$w_{0\text{-PI}} = d_0/3$

Solve for $V = V_{\mathrm{PI}}$

Model Development

Load-Deflection Model

Assumptions

- As the deflection of membrane is very small compared to its side-length, the *forces on the membrane are assumed to always act perpendicular* to the diaphragm surface.

- *The deflection of the membrane is assumed to be a quasistatic process*, i.e., the dynamic effects due to its motion (such as inertia force, damping force, etc) are not considered in this analysis.

Load-Deflection Model of a Circular Membrane

Load-deflection model for circular membrane, subject to **large deflection**, due to an applied **uniform external pressure** *P* :

$$\boxed{P = \left[\frac{4\sigma_0 h}{a^2} + \frac{64D}{a^4}\right] w_o + \left[\frac{128\alpha D}{h^2 a^4}\right] w_o^3}^{\,*} \qquad (3)$$

Combined Stiffness due to Bending and residual stress

Stiffness of the membrane due to the nonlinear stretching

$h \longrightarrow$ Membrane Thickness

$D = \dfrac{\tilde{E}h^3}{12(1-\nu^2)} \longrightarrow$ Flexural rigidity

$\tilde{E} = \dfrac{E}{1-\nu^2} \longrightarrow$ Effective Young's modulus

$a \longrightarrow$ Membrane radius

$\sigma_0 \longrightarrow$ Residual Stress

$E \longrightarrow$ Young's modulus

$\nu \longrightarrow$ Poisson ratio

$\alpha = \dfrac{7505 + 4250\nu - 2791\nu^2}{35280} \longrightarrow$ Poisson ratio dependent empirical parameter

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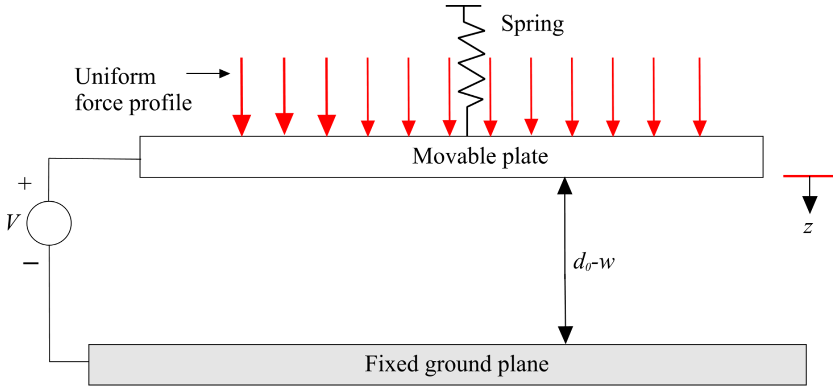
* S. Timoshenko and S. Woinowsky-Krieger, *Theory of Plates and Shells*, McGraw-Hill Book Company New York, 1959, pp. 397-428.
M. Gad-El-Haq (Editor), MEMS Handbook, Second Edition, 2002, CRC Press, Boca Raton, pp. 3-1-3-4.

Model Development

Electrostatic Pressure

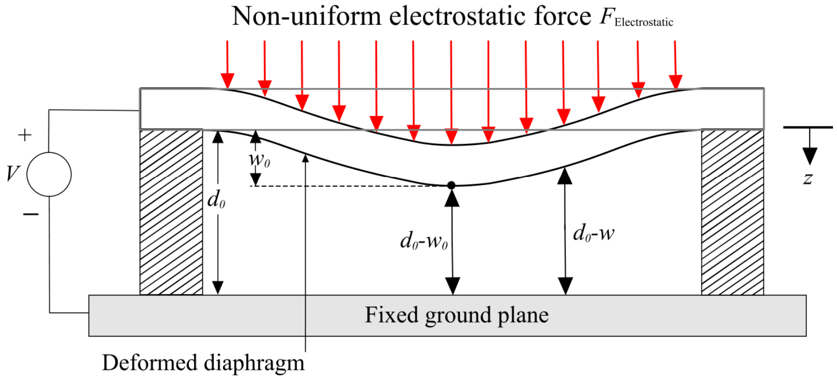
Electrostatic Force Profile



(a) Parallel plate acutator

Electrostatic force profile in a parallel plate geometry:

Uniform but Non-Linear



(b) Clamped diaphragm actuator

Electrostatic force profile in a deformed clamped square diaphragm:

Non-Uniform and Non-Linear

- No known solution for Load-Deflection Model exist with non-linear non-uniform electrostatic force

Solution Approach for Non-Linear Non-Uniform Force

Assume a **Piston-like motion** of the Diaphragm, CMUT can be approximated as **Parallel Plate Actuator**

Develop
Expression for Electrostatic Force
Uniform But Non-Linear

Linearize
Electrostatic Force using
Taylor Series Expansion

Uniform Linear Electrostatic Force

Capacitance: Undeflected Diaphragm

- Capacitance of undeflected diaphragm:

$$\begin{aligned} C &= C_0\left(1 + C_{\mathrm{ff}}\right) \\ &= \frac{a^2\pi\varepsilon_r\varepsilon_0}{d_0}\left\{1 + \frac{2d_0}{\pi\varepsilon_r a}\left[\ln\left(\frac{a}{2d_0}\right) + (1.41\varepsilon_r + 1.77) + \frac{d_0}{a}(0.268\varepsilon_r + 1.65)\right]\right\} \end{aligned} \quad * \qquad (4)$$

$C_0 = \varepsilon_0\varepsilon_r\pi a^2/d_0$ → Parallel plate capacitance (5)

$C_{\mathrm{ff}} \cong \frac{2d_0}{\pi\varepsilon_r a}\left[\ln\left(\frac{a}{2d_0}\right) + (1.41\varepsilon_r + 1.77) + \frac{d_0}{a}(0.268\varepsilon_r + 1.65)\right]$ → Fringing Field Factor (6)

$\varepsilon_0 = 8.854 \times 10^{-14}$ → Dielectric Permittivity of free space
$\varepsilon_r = 1$ (for air) → Relative Permittivity of the medium (7)

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* W. C. Chew and J. A. Kong, “Effects of Fringing Fields on the Capacitance of Circular Microstrip Disk”, *IEEE Transactions on Microwave Theory and Techniques*, Vol. 28, No. 2, pp. 98-104, Feb. 1980.

Capacitance: Deflected Diaphragm

- Capacitance of deflected diaphragm for any deflection w:

$$C_{\text{PP-Piston}} \cong \frac{a^2 \pi \varepsilon_r \varepsilon_0}{d}\left\{1+\frac{2d}{\pi\varepsilon_r a}\left[\ln\left(\frac{a}{2d}\right)+(1.41\varepsilon_r+1.77)+\frac{d}{a}(0.268\varepsilon_r+1.65)\right]\right\} \qquad (8)$$

where, $d = d_0 - w$, is the separation between the diaphragm and the backplate

Assumption made	Piston-like motion of the diaphragm (Parallel Plate Approximation)

- $\varepsilon_r = 1$ (for air), substituting in (8) the Capacitance expression becomes:

$$C_{\text{pp-piston}} \cong \frac{a^2 \pi \varepsilon_0}{d_0-w}\left\{1+\frac{2(d_0-w)}{\pi a}\left[\ln\left(\frac{a}{2(d_0-w)}\right)+\frac{1.918(d_0-w)}{a}+3.18\right]\right\} \qquad (9)$$

Electrostatic Force

- The developed *Electrostatic Force* after applying a bias voltage V

$$F_{\text{electrostatic}} = -\frac{d}{dz}\left(\frac{1}{2}C_{\text{pp-piston}}V^2\right) = \left[\frac{\pi a^2}{2(d_o - w)^2} + \frac{a}{d_o - w} - 1.918\right]\varepsilon_0 V^2 \qquad (10)$$

- After Linearizing using Taylor series expansion method about the zero deflection point and rearranging the terms:

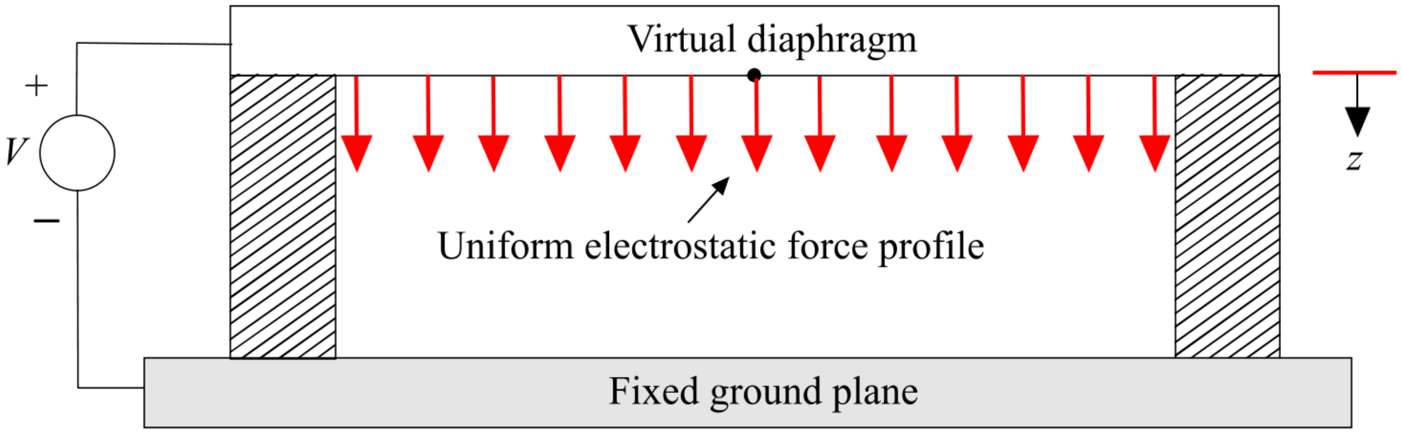
$$F_{\text{electrostatic}} = \varepsilon_0 \pi a^2 V^2 \left[\frac{1}{2d_0^2} + \frac{1}{\pi a d_0} + \frac{1.918}{\pi a^2}\right] + \varepsilon_0 \pi a^2 V^2 \left[\frac{1}{d_0^3} + \frac{1}{\pi a d_0^2}\right] w \qquad (11)$$

Electrostatic Pressure

- Replacing $w = w_0$, the *Electrostatic Pressure* is given by,

$$P_{\text{electrostatic}} = \frac{F_{\text{electrostatic}}}{A} = \varepsilon_0 V^2 \left[\frac{1}{2d_0^2} + \frac{1}{\pi a d_0} + \frac{1.918}{\pi a^2} \right] + \varepsilon_0 V^2 \left[\frac{1}{d_0^3} + \frac{1}{\pi a d_0^2} \right] w_0 \tag{12}$$

Load-deflection model due to electrostatic pressure
Yields uniform linear electrostatic pressure profile



Virtual planar diaphragm after linearizng the electrostatic force about the zero deflection point of the diaphragm center

Model Development

Pull-in Voltage

Pressures on Diaphragm at Pull-in Equilibrium

- For Parallel Plate Actuator, diaphragm travel at pull-in

$$w_{0\text{–PI}} = d_0/3 \qquad (1)$$

- At *pull-in equilibrium,*

1. Electrostatic Pressure $P_{\text{PI-electrostatic}}$ (from electrostatic pressure expression) [after substituting $w_0 = d_0/3$ in (12)]:

$$\boxed{P_{\mathrm{PI-electrostatic}} = \varepsilon_0 V_{\mathrm{PI}}^2 \left[\frac{1}{2d_0^2} + \frac{1}{\pi a d_0} + \frac{1.918}{\pi a^2}\right] + \varepsilon_0 V_{\mathrm{PI}}^2 \left[\frac{1}{d_0^3} + \frac{1}{\pi a d_0^2}\right]\left(\frac{d_0}{3}\right)} \qquad (13)$$

2. Elastic Restoring Pressure $P_{\text{PI-elastic}}$ (from load-deflection model) [after substituting $w_0 = d_0/3$ in (3)]

$$\boxed{P_{\mathrm{PI-elastic}} = \left[\frac{64D}{a^4} + \frac{4\sigma h}{a^2}\right]\left(\frac{d_0}{3}\right) + \frac{128\alpha D}{h^2 a^4}\left(\frac{d_0}{3}\right)^3} \qquad (14)$$

Pull-in Voltage

- At *pull-in equilibrium*, the **electrostatic pressure** is just counterbalanced by the **elastic restoring pressure**

$$\boxed{P_{\mathrm{PI-electrostatic}} = P_{\mathrm{PI-elastic}}} \qquad (15)$$

- (13) and (14) can be solved simultaneously to yield the closed-form model for the pull-in voltage V_{PI} for the circular diaphragm as:

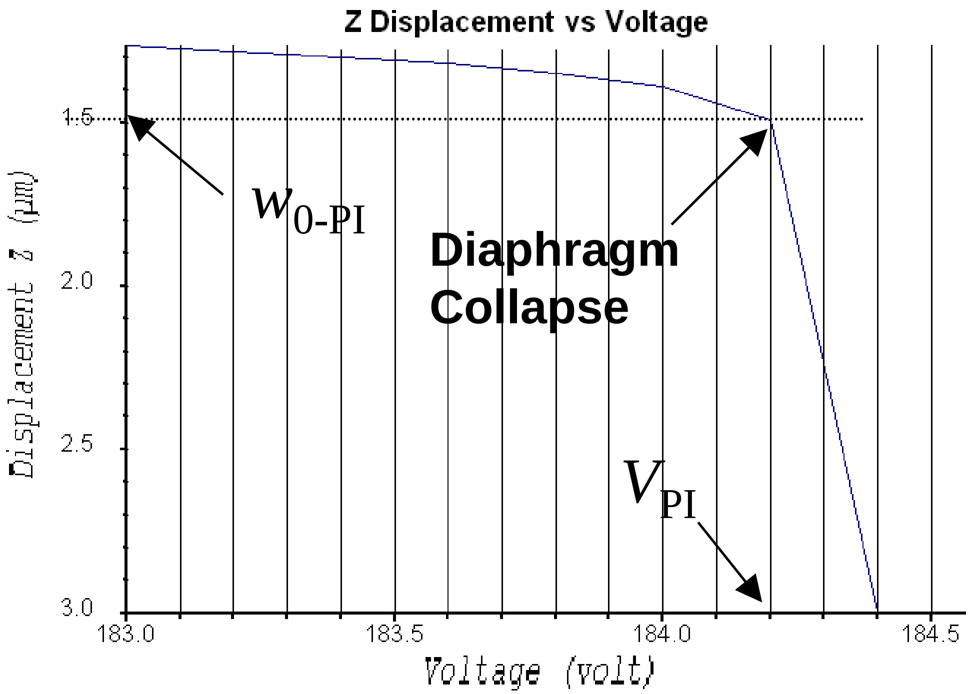
$$\boxed{V_{\mathrm{PI}} = \sqrt{\frac{\left[\frac{64D}{a^4} + \frac{4\sigma h}{a^2}\right]\left(\frac{d_0}{3}\right) + \frac{128\alpha D}{h^2 a^4}\left(\frac{d_0}{3}\right)^3}{\varepsilon_0\left[\frac{5}{6d_0^2} + \frac{4}{3\pi a d_0} + \frac{1.918}{\pi a^2}\right]}}} \qquad (16)$$

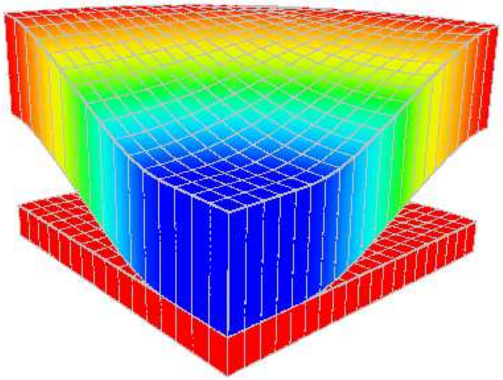
Model Validation

Finite Element Analysis Using IntelliSuite

Table I. Device Specifications Used in FEA Simulation

Parameter	Unit	Value
Diaphragm Radius (a)	µm	250
Diaphragm Thickness (h)	µm	1-3
Airgap Thickness (d_0)	µm	2-3
Young's Modulus (E)	GPa	169
Poisson Ratio (ν)	-	0.3





(Right) Screen capture of the diaphragm collapse after carrying out a 3-D electromechanical FEA using InteilliSuite. (h = 3 µm, d_0= 3 µm, σ_0 = 100 MPa).

Model Validation

Table II. Pull-in Voltage Comparison

Diaphragm thickness, h =1 μm, Airgap thickness, d_0 =2 μm

Residual stress (MPa)	(V)			Δ%	
	New Model	Ref. [14]	FEA	FEA - New model	FEA - Ref. [14]
0	11.03	9.89	12.4	10.98	20.24
100	49.24	50.48	49.9	1.3	1.16
250	76.69	77.88	76.86	0.22	1.32
350	90.46	91.56	90.4	0.074	1.28

Diaphragm thickness, h =1 μm, Airgap thickness, d_0 =3 μm

0	22.38	18.17	28.04	20.17	35.2
100	90.88	92.83	91.4	0.56	1.46
250	141.06	143.07	139.9	0.82	2.26
350	166.3	168.2	164.4	1.15	2.31

Ref [14] P. M. Osterberg, "Electrostatically Actuated Microelectromechanical Test Structures for Material property Measurements," *Ph.D. Dissertation*, MIT, Cambridge, MA, 1995, pp. 52-87.

Model Validation

Table III. Pull-in Voltage Comparison

Diaphragm thickness, h =2 μm, Airgap thickness, d_0 =3 μm

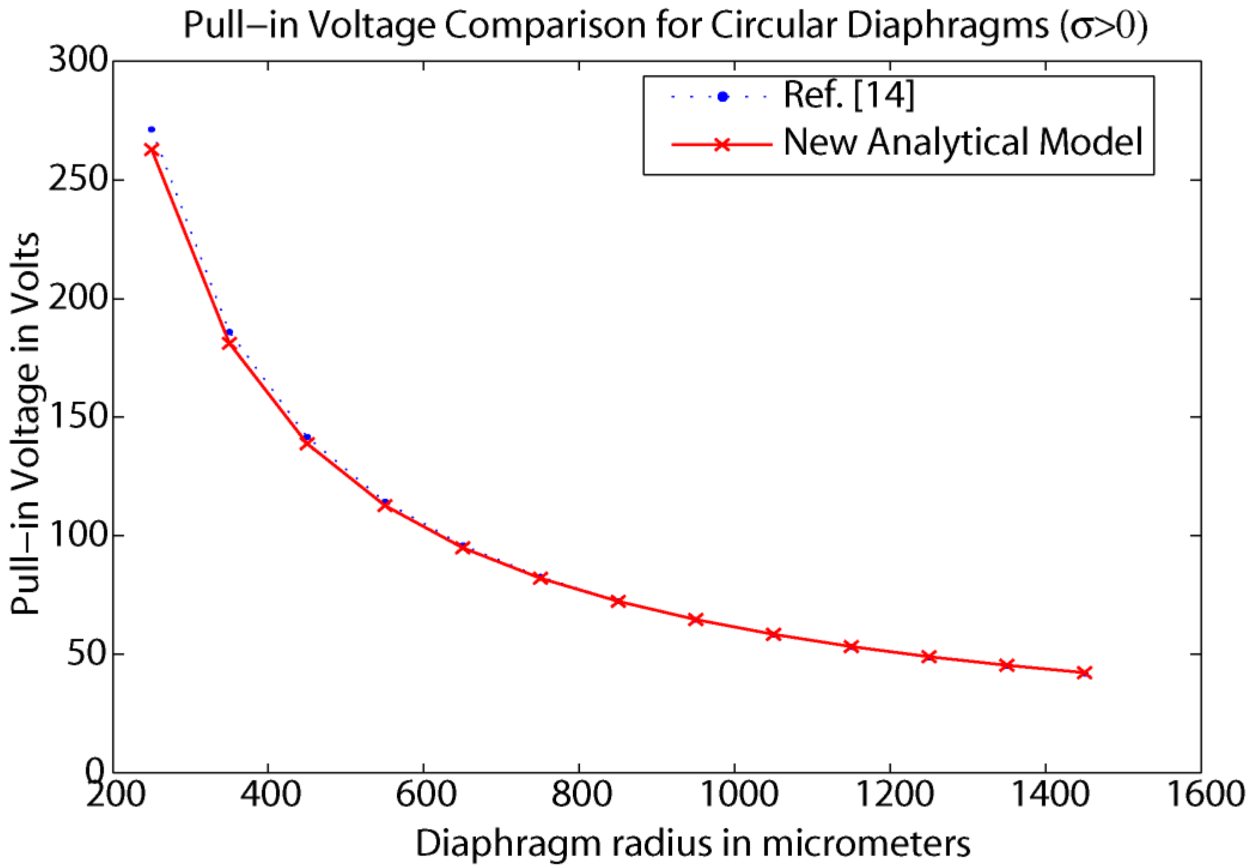
Residual stress (MPa)	(V)			Δ%	
	New Model	Ref. [14]	FEA	FEA - New model	FEA - Ref. [14]
0	55.03	51.39	57.6	4.45	10.77
100	136.18	141.18	138.4	1.6	2.0
250	204.5	211.24	205.6	0.53	2.74
350	239.45	246.53	239.8	0.15	2.8
Diaphragm thickness, h =3 μm, Airgap thickness, d_0 =3 μm					
0	98.03	94.42	97.7	0.33	3.36
100	181.34	188.14	184.2	1.55	2.14
250	260.38	271.20	263.4	1.14	2.96
350	301.78	313.77	304.4	0.85	3.07

Ref [14] P. M. Osterberg, “Electrostatically Actuated Microelectromechanical Test Structures for Material property Measurements,” *Ph.D. Dissertation*, MIT, Cambridge, MA, 1995, pp. 52-87.

Model Validation

Pull-in Voltage *vs* Diaphragm Radius (σ_0>0)

(Diaphragm thickness, h =3 µm, Airgap thickness, d_0 =3 µm , Residual stress σ_0=250 MPa)

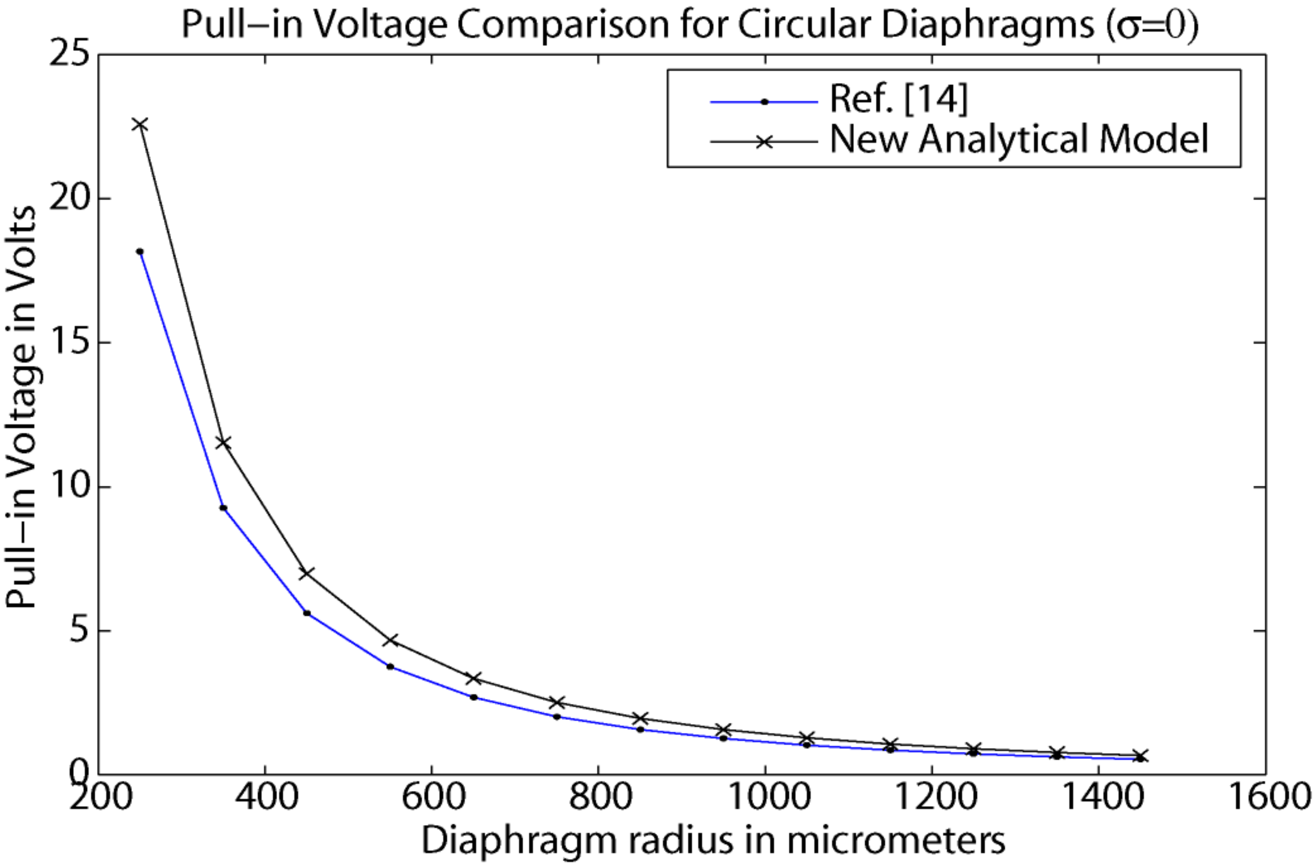


- **Both the models merge at large diaphragm radius**

Model Validation

Pull-in Voltage *vs* Diaphragm Radius (σ_0=0)

(Diaphragm thickness, h =3 μm, Airgap thickness, d_0 =3 μm , Residual stress σ_0=0)



- **Both the models merge at large diaphragm radius**

Model Limitations

The new analytical method can provide very good approximation of pull-in voltage for the following limited cases:

- Both the electrodes are required to be parallel prior to any electrostatic actuation.
- The gap between the clamped diaphragm and the backplate should be small enough so that the Taylor series expansion about the zero deflection point doesn’t introduce any significant error.
- The lateral dimensions of the diaphragm are required to be very large compared to the diaphragm’s thickness and the airgap.

Conclusions

- ☞ A **highly accurate closed-form model** for the **pull-in voltage** of a clamped circular diaphragm CMUT device is presented.
- ☞ The model is **simple, easy to use** and **fast**, and takes into account both the **non-linear stretching due to large deflection** and the **non-linearity of electrostatic field**.
- ☞ Excellent agreement with 3-D electromechanical FEA using IntelliSuite with a **maximum deviation of 1.6%** for diaphragms with residual stress
- ☞ Can also be easily extended to a *Clamped-edge Square Membrane*.
- ☞ Besides CMUTs, the Model will also be useful in **ensuring safe and efficient operation** of
    - MEMS capacitive type pressure sensors
    - MEMS based microphones
    - Touch mode pressure sensors
    - Other application areas where electrostatically actuated circular diaphragms are used.

Thanks for your Patience