



# Two Dimensional Wavelet and its Application

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# Outline

- Problem Statement
- Fourier Transform
- Short Time Fourier Transform
- One Dimensional Wavelet
- Two Dimensional Wavelet
- Two Dimensional Wavelet in Application
- Conclusion

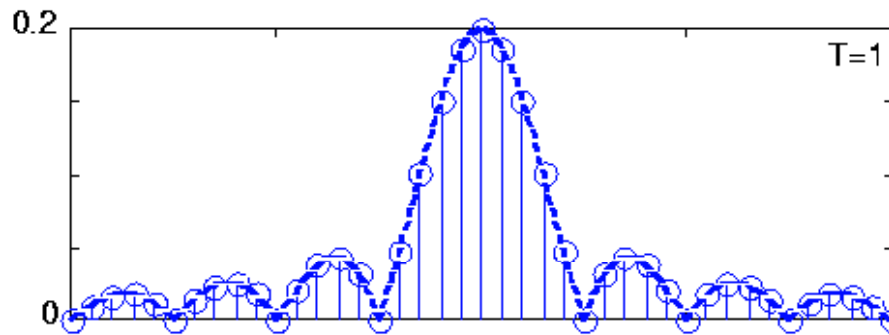


# Problem Statement

- What is wavelet?
- How and where can I use it?

# Fourier Transform

- In early 1800 was introduced by Josef Fourier
- It is good for analyzing periodic functions.
- Has a sinusoidal basis.
- Just contains frequency domain information.
- Time domain information are lost.



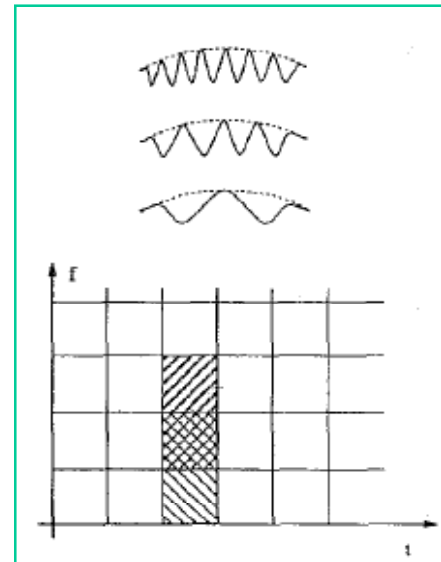
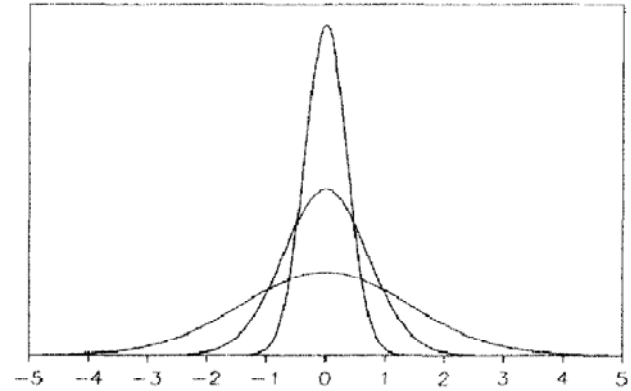


# Short Time Fourier Transform

- In 1964, Gabor introduced windowed Fourier atoms.
- The support area of the basis has reduced using windows.

$$g_{\alpha}(t) = \frac{1}{2\sqrt{\pi\alpha}} e^{-\frac{t^2}{4\alpha}}$$

$$g_{b,\omega}(t) = e^{i\omega t} g_{\alpha}(t - b)$$





# How does wavelet work?

- A row of an image [ 8 4 1 3]
- Representing in the *Haar basis*

Resolution	Average	Detail Coefficients
4	[8 4 1 3]	
2	[6 2]	[2 -1]
1	[4]	[2]

- The wavelet transform is [4 2 2 -1].
- Two Function in wavelet transform,
  - Scaling Function (Average, Lowpass Filter)
  - Wavelet Function (Detail, Highpass Filter)

# 1D Haar Wavelet

- Haar Scaling Function

- $V_0$  the space of all function  $\sum_{k \in \mathbb{Z}} a_k \phi(x - k)$

- $V_j$  the space of all function  $\sum_{k \in \mathbb{Z}} a_k \phi(2^j x - k)$

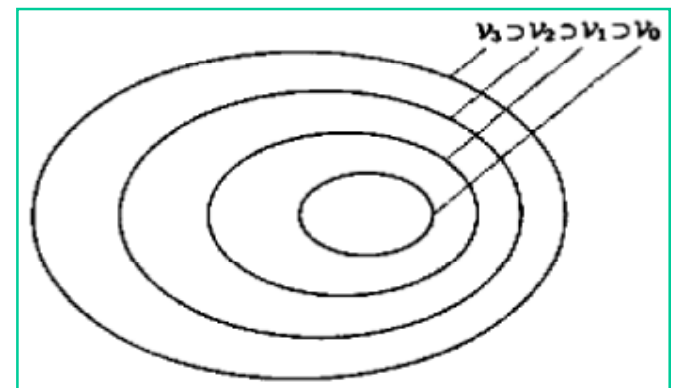
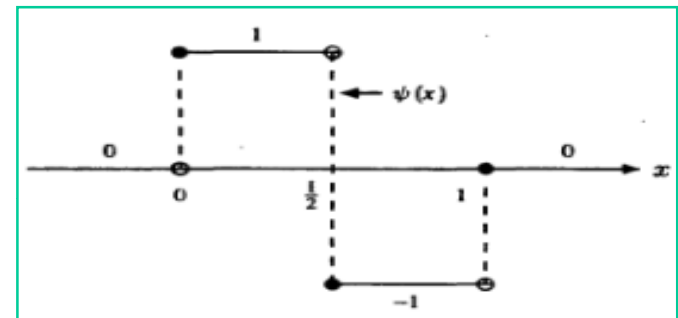
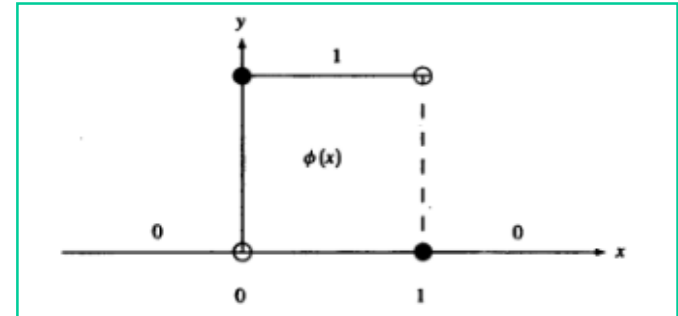
- Haar Wavelet Function

- $W_0$  the space of all function  $\sum_{k \in \mathbb{Z}} a_k \psi(x - k)$

- $W_j$  the space of all function  $\sum_{k \in \mathbb{Z}} a_k \psi(2^j x - k)$

- $W_j$  is the orthogonal complement of  $V_j$  in  $V_{j+1}$ .

- Multiresolution Concept





# 1D Haar Wavelet

$$\mathcal{I}(x) = c_0^2 \phi_0^2(x) + c_1^2 \phi_1^2(x) + c_2^2 \phi_2^2(x) + c_3^2 \phi_3^2(x), \quad \mathcal{I}(x) = c_0^1 \phi_0^1(x) + c_1^1 \phi_1^1(x) + d_0^1 \psi_0^1(x) + d_1^1 \psi_1^1(x)$$

$$\begin{aligned} \mathcal{I}(x) &= 8 \times \text{[Step function: 1 on } [0, 1], 0 \text{ elsewhere]} \\ &+ 4 \times \text{[Step function: 1 on } [1, 2], 0 \text{ elsewhere]} \\ &+ 1 \times \text{[Step function: 1 on } [2, 3], 0 \text{ elsewhere]} \\ &+ 3 \times \text{[Step function: 1 on } [3, 4], 0 \text{ elsewhere]} \end{aligned}$$

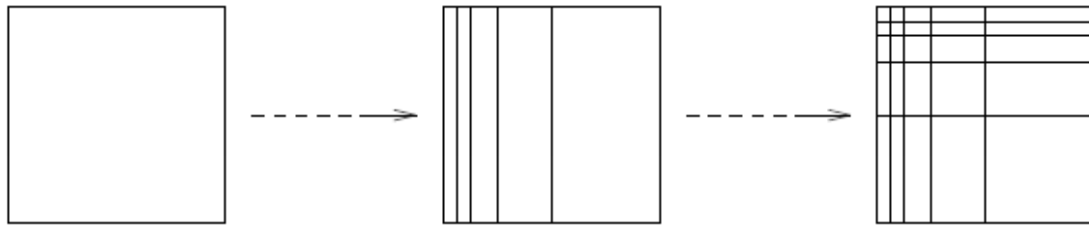
$$\begin{aligned} &= 6 \times \text{[Step function: 1 on } [0, 2], 0 \text{ elsewhere]} \\ &+ 2 \times \text{[Step function: 1 on } [2, 4], 0 \text{ elsewhere]} \\ &+ 2 \times \text{[Step function: 1 on } [0, 1], -1 \text{ on } [1, 2], 0 \text{ elsewhere]} \\ &+ -1 \times \text{[Step function: 1 on } [2, 3], -1 \text{ on } [3, 4], 0 \text{ elsewhere]} \end{aligned}$$

$$\begin{aligned} \mathcal{I}(x) &= c_0^0 \phi_0^0(x) + d_0^0 \psi_0^0(x) + d_1^0 \psi_1^0(x) + d_1^1 \psi_1^1(x) \\ &= 4 \times \text{[Step function: 1 on } [0, 4], 0 \text{ elsewhere]} \\ &+ 2 \times \text{[Step function: 1 on } [0, 2], -1 \text{ on } [2, 4], 0 \text{ elsewhere]} \\ &+ 2 \times \text{[Step function: 1 on } [0, 1], -1 \text{ on } [1, 2], 0 \text{ elsewhere]} \\ &+ -1 \times \text{[Step function: 1 on } [2, 3], -1 \text{ on } [3, 4], 0 \text{ elsewhere]} \end{aligned}$$

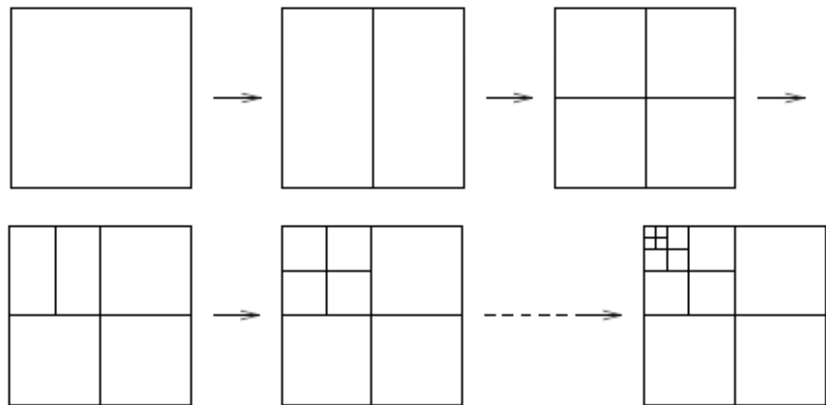


# 2D Haar Wavelet

- Two approaches for dealing with 2D signals.
  - Standard decomposition



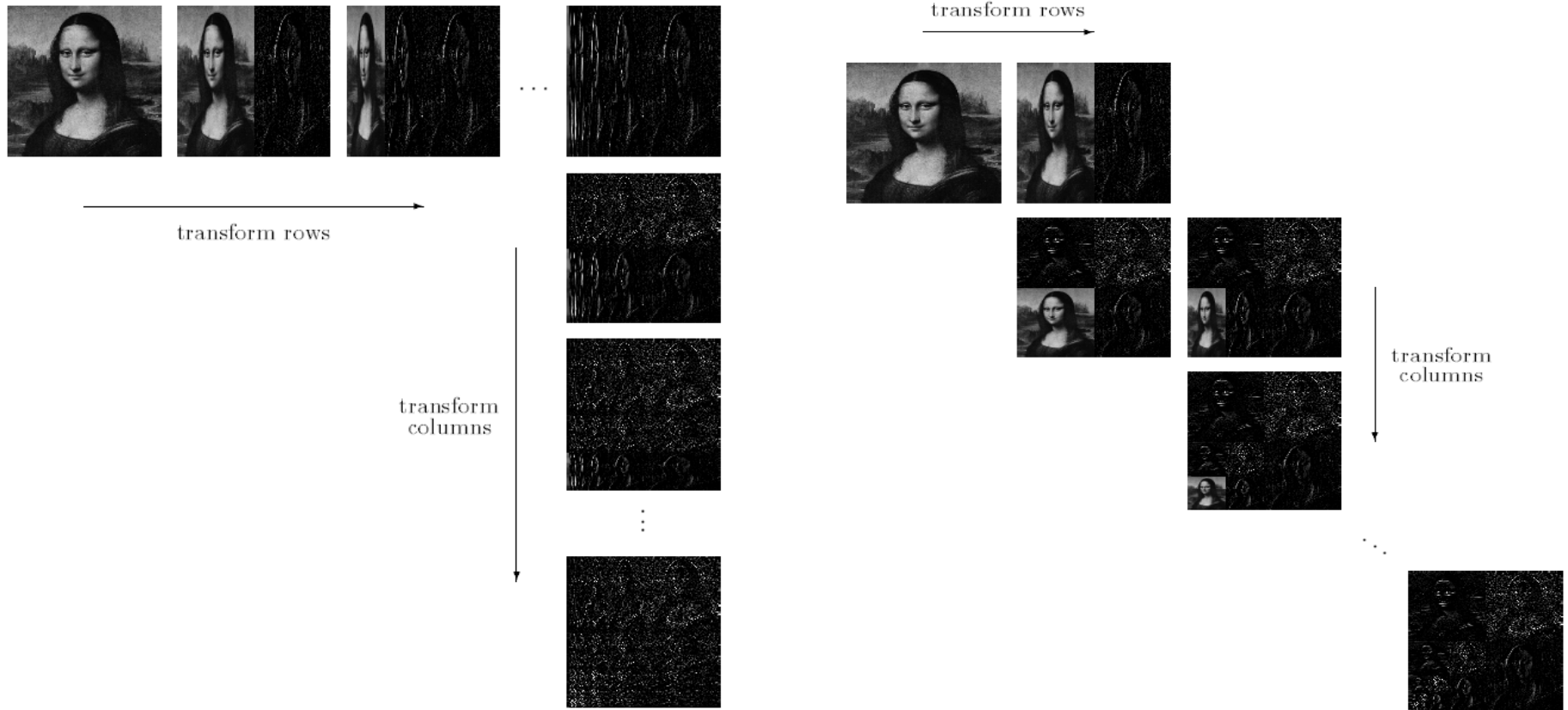
- Non-standard decomposition



- Advantages and disadvantages
  - S.D. is simple.
  - N.S.D is more efficient in computation.



# 2D Haar Wavelet





# 2D Haar Wavelet

- Non-standard Approach
  - Scaling Function

$$\Phi(x, y) = \varphi(x)\varphi(y),$$

- Wavelet Functions

$$\Psi_1(x, y) = \varphi(x)\psi(y),$$

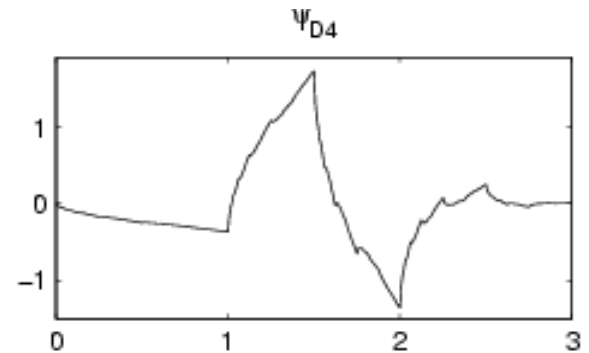
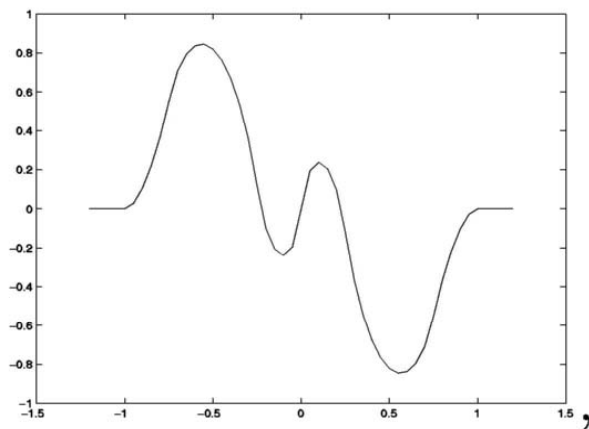
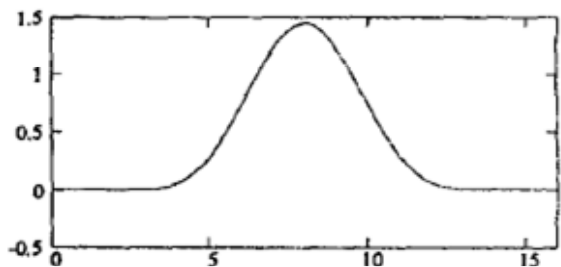
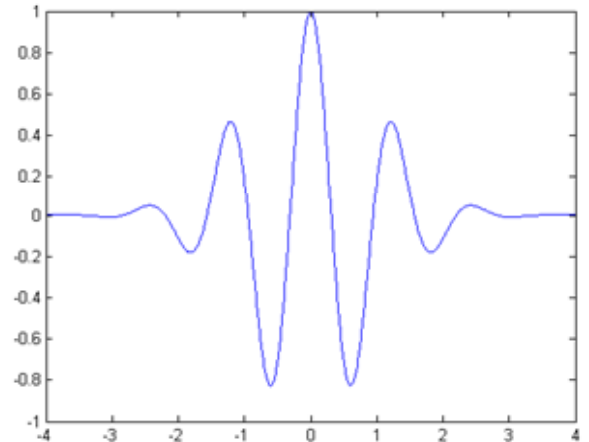
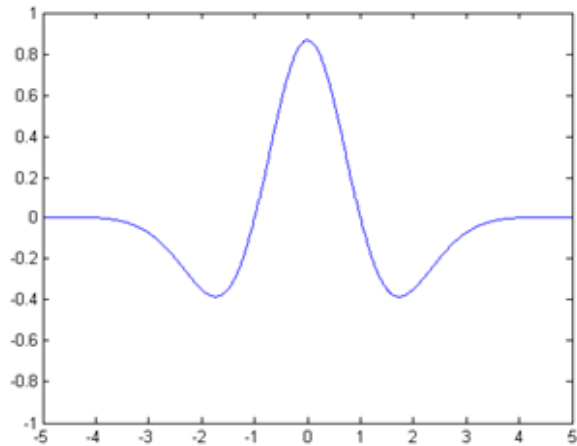
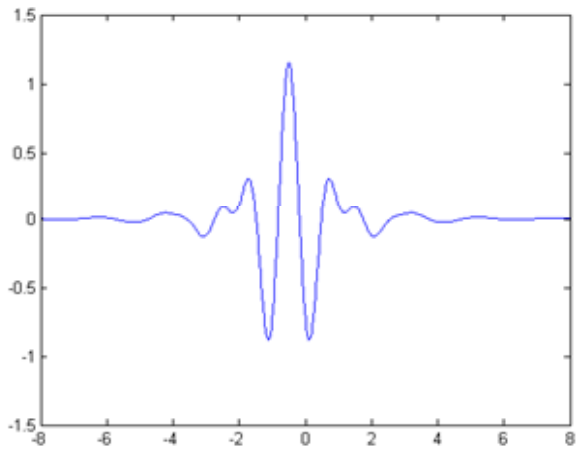
$$\Psi_2(x, y) = \psi(x)\varphi(y),$$

$$\Psi_3(x, y) = \psi(x)\psi(y).$$

LL <sub>2</sub>	HL <sub>2</sub>	HL <sub>1</sub>
LH <sub>2</sub>	HH <sub>2</sub>	
LH <sub>1</sub>		HH <sub>1</sub>

# Wavelet Functions

- Different purposes, different wavelets,





# 2D Wavelet in Application

- Filtering
- Image Comparison
- Fault Prognosis
- Classification
- Document Analysis
- Hardware Implementation
- Feature Extracting(\*)

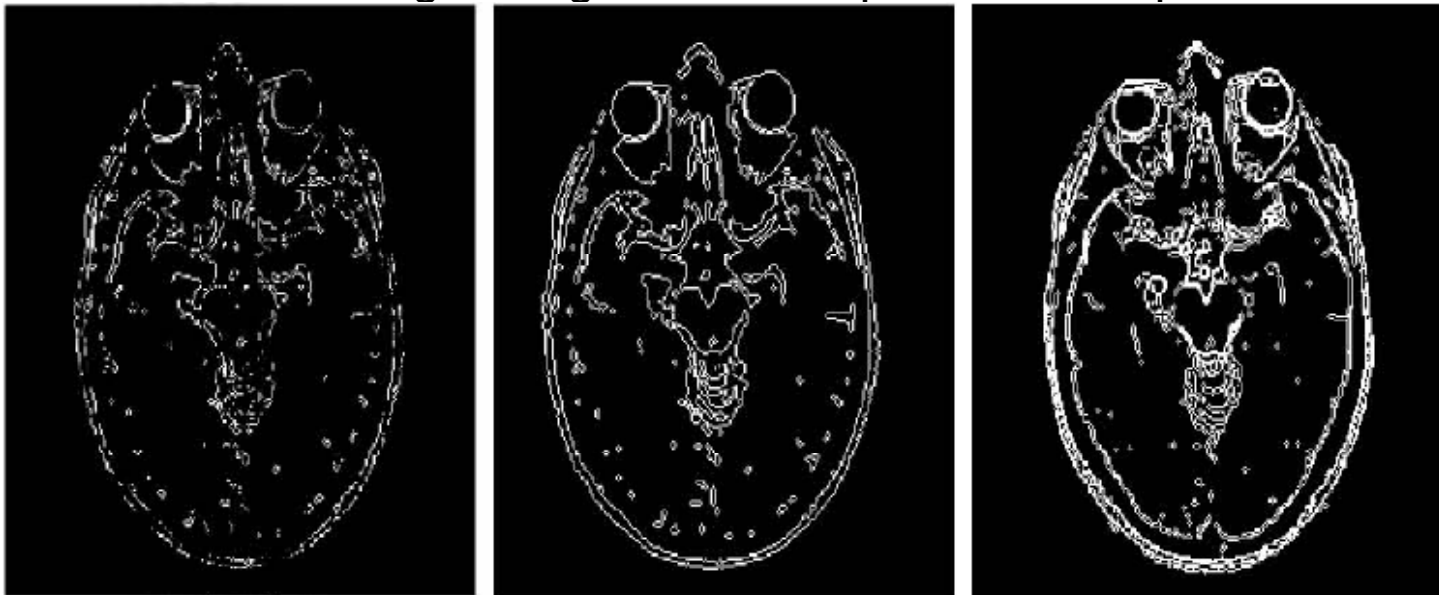


# Feature Extraction

- “Wavelet Segmentation for Fetal Ultrasound Images”, 2001
  - Efficient algorithm for segmentation
  - Decomposes the input image into a multiresolution space
  - Feature vector for each pixel in the wavelet domain including
    - Mean
    - Variance
  - Fuzzy C-means Clustering for segmentation
  - Less noisy segmented image

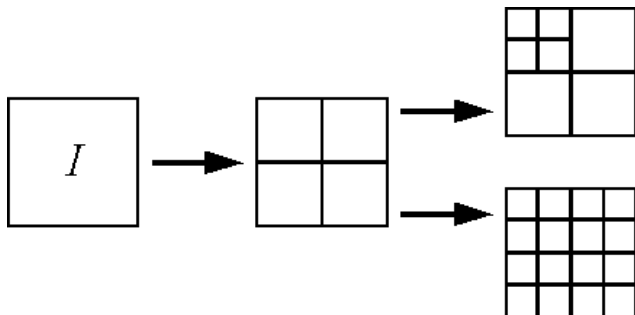
# Feature Extraction

- “Efficient Segmentation in MRI Applying Discrete Wavelet Transform”, 2005
  - Main goal is a better identification of abrupt changes without increasing the presence of noise.
  - A sliding window scanning the image and calculating the wavelet coefficients
  - Use of ICA to extract *Independent* wavelet descriptors
  - Kohonen’s Self Organizing Feature Map as the unsupervised classifier



# Feature Extracting

- “Choosing Best Basis in Wavelet Packets for Fingerprint Matching” 2004
  - Wavelet packets: An over-complete version of regular wavelet decomposition
  - The ridge-valley structure of fingerprint
  - Energy in a sub-band reaches a local MAX when the scale of wavelet packet basis best matches the spatial freq. of the ridge-valley struct.
  - Energy of different sub-bands gives information regarding both the edge spatial frequency as well as the ridge orientation.
  - A tree selection algorithm for selecting the best informative branch.





# Feature Extracting

- “Advanced System for Automating Eddy-Current Nondestructive Evaluation”, 2000
  - Measuring a metal’s impedance using c-scanning mode, which gives a matrix format data, like an image.
  - Images divided into sub-images, wavelet decomposition is applied on them.
  - Average, standard deviation of each sub-images as the feature vector.
  - A neural network as a classifier to classify sub-surface flaws.



# Conclusion

- About five books and 17 papers
- Most of the papers were published after 2000
- More focus on feature extraction application
- Different methods for feature extraction