



Edge Detection using Mathematical Morphology



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June 15, 2007

Outline

- Introduction to Mathematical Morphology
- The Structuring Element
- Basic and Composite Operations
- Morphological Edge Detection
- Reduced Noise Morphological Edge Detection
- Other Edge Detectors
- Real-Time Edge Detection

Mathematical Morphology

- Developed by Matheron and Serra at L'Ecole des Mines in Paris.
- Uses set theory image analysis
- Can be used to find boundaries, skeletons, etc.
- Can be used for many pre and post image processing techniques
- Relies on two basic operations used to shrink and expand image features
- Originally used on binary images



Mathematical Morphology

- Two Basic Operations
 - Dilation (expands features)
 - Erosion (shrinks features)
- Composite Relations
 - Opening
 - Closing
- Operations
 - Edge Detection
 - Thinning
 - Thickening
 - Etc.

Structuring Element

- Small Binary Image
 - 3x3, 5x5, 7x7, etc.
- Swept over image
- Typical elements
 - Square
 - Disk
 - Ring
- Origin used as index pixel

1	1	1
1	1	1
1	1	1

Square (3x3)

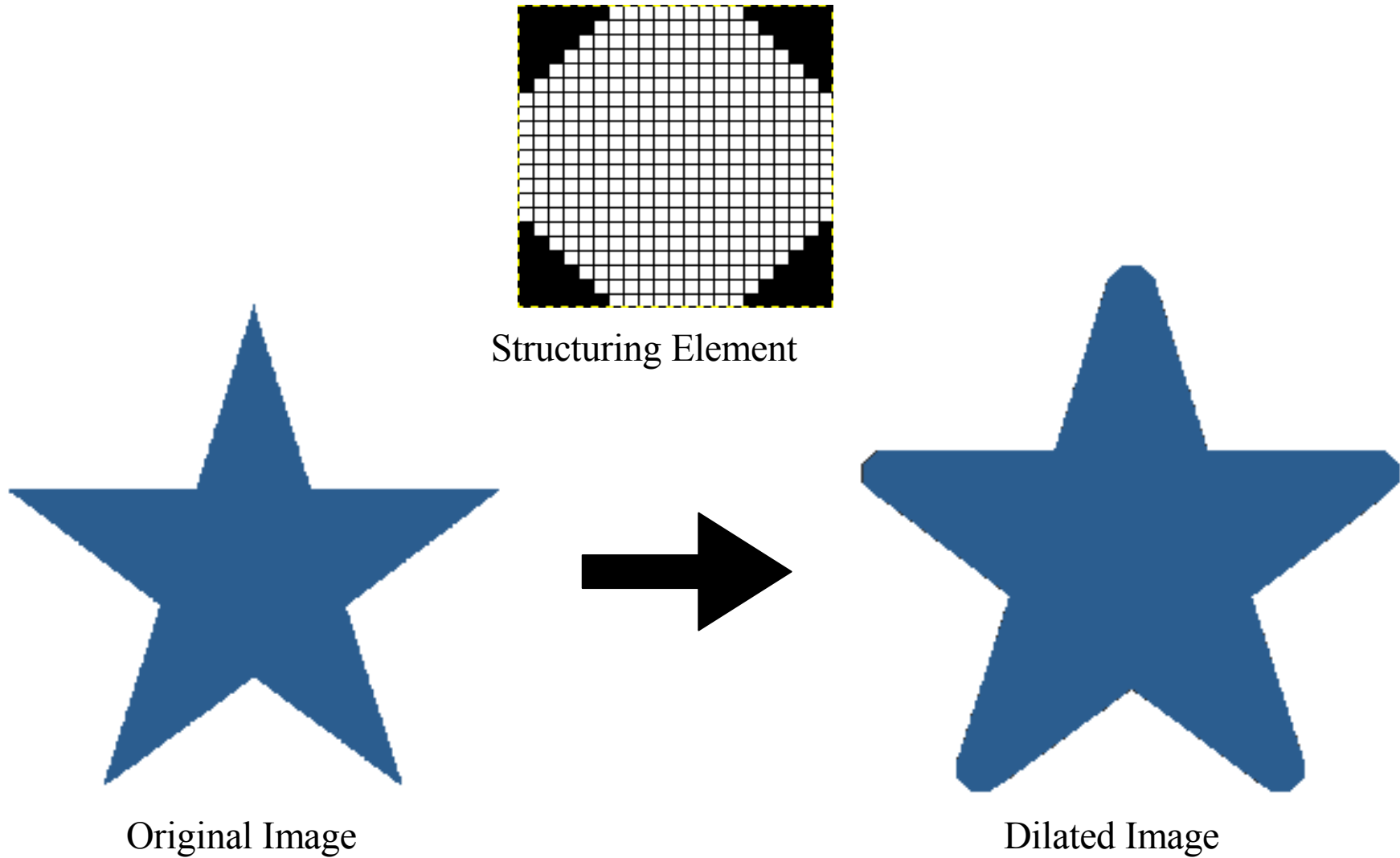
	1	1	1	
1				1
1				1
1				1
	1	1	1	

Ring (5x5)

	1	1	1	
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
	1	1	1	

Disk (5x5)

Dilation



Dilation

- Expands features
- Structuring element swept over image
- Index pixel set to maximum found within structuring element

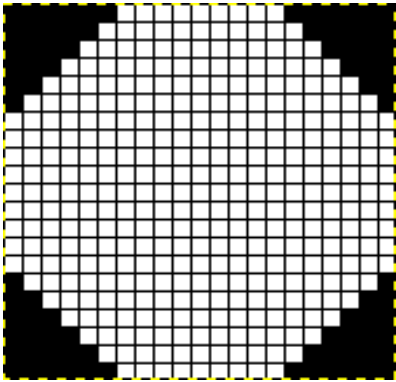
$$A \oplus B = \max_{[j,k] \forall B} \{ a[m-j, n-k] + b[j, k] \}$$

Where,

A – Original Image

B – Structuring Element

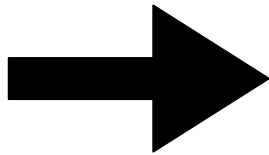
Erosion



Structuring Element



Original Image



Eroded Image

Erosion

- Reduces features
- Structuring element swept over image
- Index pixel set to minimum found within structuring element

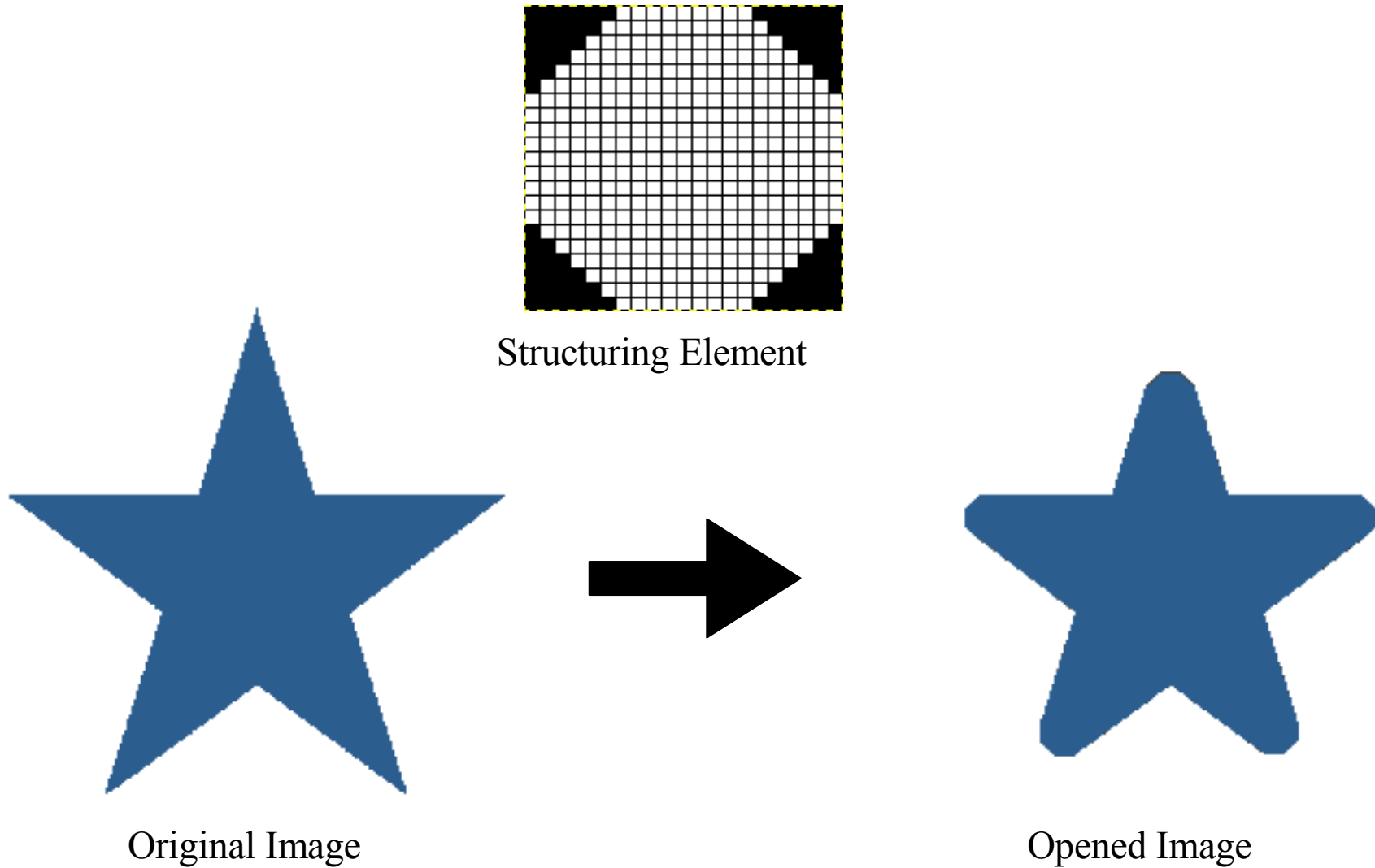
$$A \ominus B = \min_{[j,k] \in B} \{ a[m-j, n-k] - b[j, k] \}$$

Where,

A – Original Image

B – Structuring Element

Opening



Opening

- Function of dilation and erosion
- Structuring element rolled along inner boundary

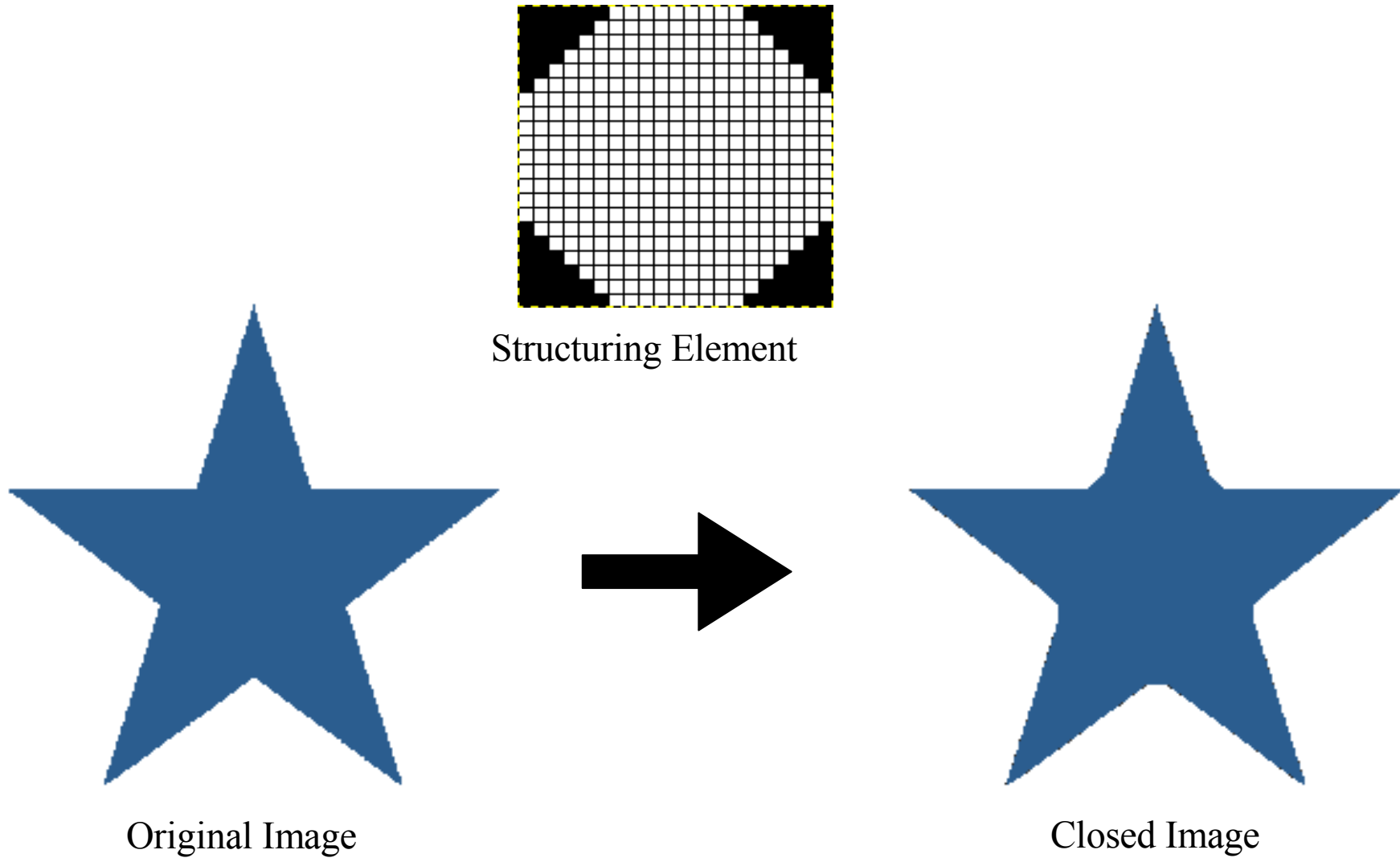
$$A \circ B = (A \ominus B) \oplus B$$

Where,

A – Original Image

B – Structuring Element

Closing



Closing

- Function of erosion and dilation
- Structuring element rolled along outer boundary

$$A \cdot B = (A \oplus B) \ominus B$$

Where,

A – Original Image

B – Structuring Element

Morphological Edge Detection

- Several edge detection techniques using morphology.
 - Erosion Residue Edge Detector
 - Dilation Residue Edge Detector
 - Morphological Gradient Edge Detector
 - Reduced noise Morphological Gradient Edge Detector

Morphological Gradient Edge Detection

- Based on eroded and dilated image
- Provides good edge detection
- Should be followed by threshold for most applications

$$E_G(A) = (A \oplus B) - (A \ominus B)$$

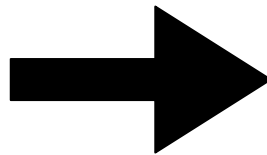
Morphological Gradient Edge Detection

	1	
1	1	1
	1	

Structuring Element



Original Image

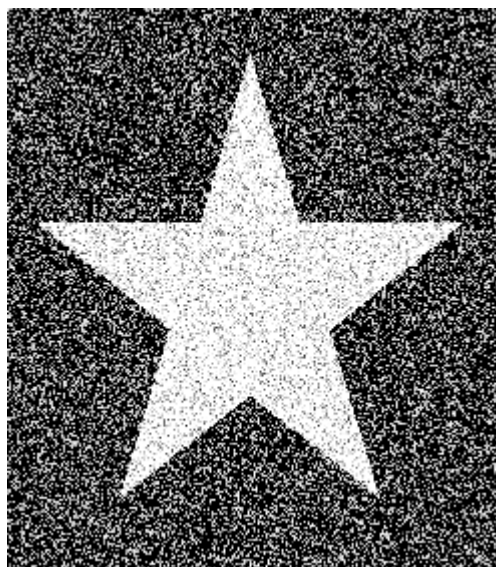


Edge Detection

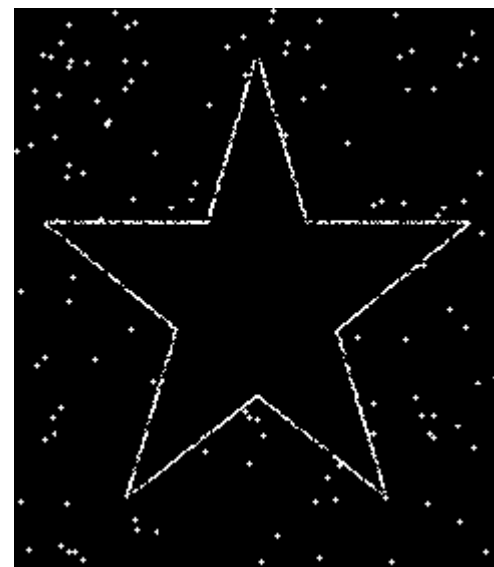
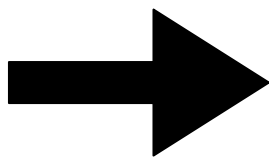
Morphological Gradient Edge Detection (noisy image)

	1	
1	1	1
	1	

Structuring Element



Original Noisy Image



Edge Detection (after thresholding)

Reduced Noise Morphological Edge Detection

- Proposed by Zhao et al. [1]
- Reduced noise
- Opening and closing to pre-process image filtering noise
- Close M to smooth

$$E_{\text{reduced-noise}}(A) = (M \cdot B) \oplus B - M \cdot B$$

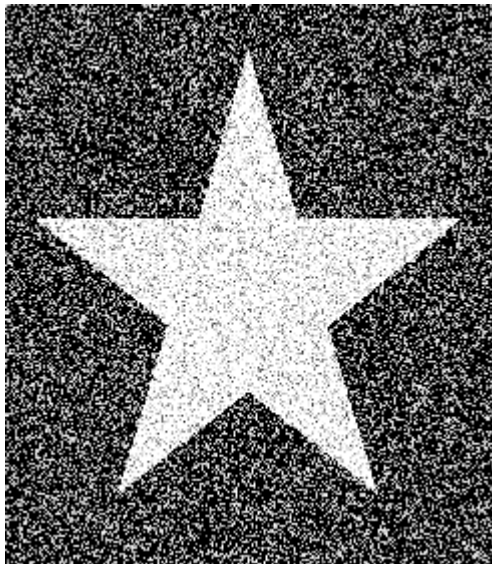
where,

$$M = (F \cdot B) \circ B$$

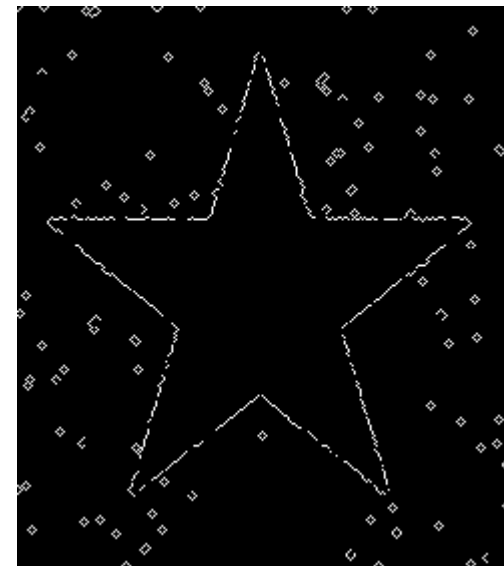
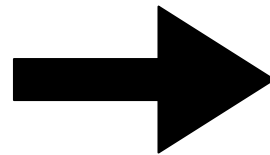
Reduced Noise Morphological Gradient Edge Detection

	1	
1	1	1
	1	

Structuring Element



Original Noisy Image



Edge Detection (after thresholding)

Other Edge Detection Algorithms

- Some very well known edge detection algorithms include
 - Prewitt
 - Sobel
 - Canny
- These techniques detect edges based on directional gradients

Prewitt and Sobel Edge Detection Algorithms

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Sobel Convolution Kernel

$$G_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Prewitt Convolution Kernel

$$G_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

The gradient magnitude is given by:

$$G = \sqrt{G_x^2 + G_y^2}$$

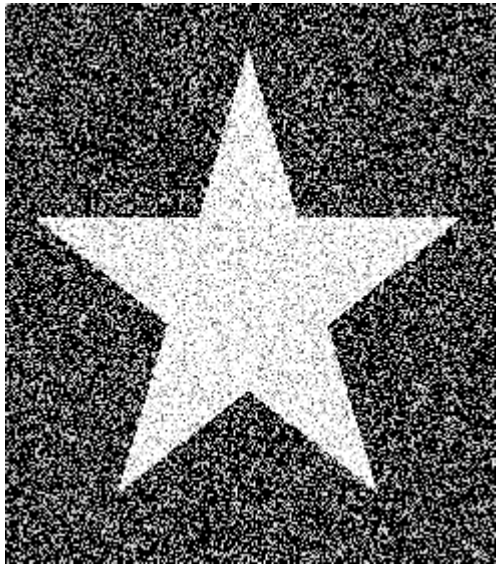
and can be approximated as follows:

$$G \approx |G_x| + |G_y|$$

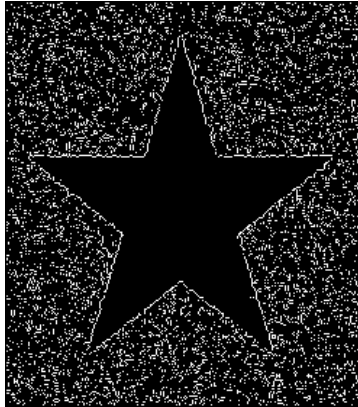
Canny Edge Detection Algorithm

- More complex than Sobel, Prewitt and Morphological Gradient
- Six Step Process
 - Gaussian Mask (noise removal)
 - Sobel Operator to find edge strengths
 - Edge directions are computed
 - Edge directions are descretized
 - Non-maxima suppression
 - Hysteresis

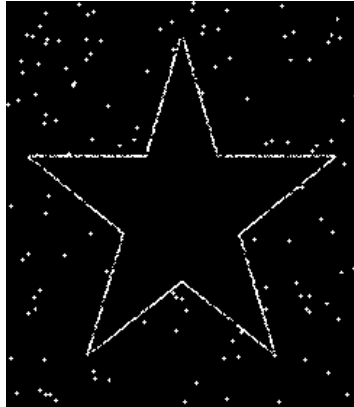
Comparison of Algorithms on a Noisy Image



Original Image



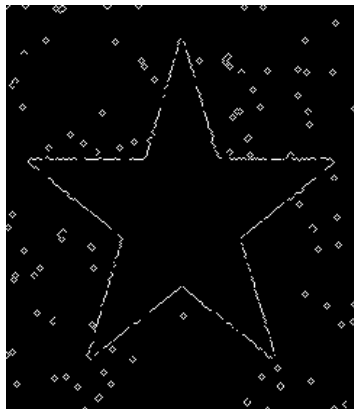
Prewitt



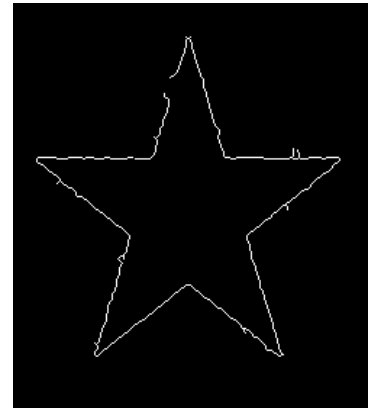
Morphological
Gradient



Sobel



Reduced Noise
Morphological
Gradient



Canny

Real Time Edge Detection

- Edge detection can be performed in real-time.
- Can easily be done using the Morphological Gradient, Sobel or Prewitt algorithms
- A storage element is essential
- Delay will be imminent
- Must be performed on gray level image information



Questions?

References

- [1] S. Aksoy, “Binary Image Analysis”, Internet:
http://cs.bilkent.edu.tr/~saksoy/courses/cs484/slides/cs484_binary.pdf, 2007, [cited 2007 Apr 1].
- [2] William Green, “Edge Detection Tutorial.” Internet:
<http://www.pages.drexel.edu/~weg22/edge.html>, 2002, [cited 2007 Apr 5].
- [3] Zhao Yu-qian, Gui Wei-hua, Chen Zhen-cheng, Tang Jing-tian and Li Ling-yun, “Medical Images Edge Detection Based on Mathematical Morphology,” in Proc. IEEE-EMBS, 2005, pp. 6492-6495.
- [4] Robyn Owens, “Mathematical Morphology.” Internet:
http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/OWENS/LECT3/node3.html, 1997, [cited 2007 June 13].
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<http://www.cee.hw.ac.uk/hipr/html/sobel.html>, 1994, [cited 2007 June 13].